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S_3 flavor symmetry and leptogenesis

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Abstract. We consider leptogenesis in a minimal S_3 extension of the standard model with an additional Z_2 symmetry in the leptonic sector. It is found that the CP phase appearing in the mass matrix of the left-handed neutrinos is the same as that for the CP asymmetries responsible for leptogenesis. Because of the discrete $S_3 \times Z_2$ flavor symmetries, the CP asymmetries are strongly suppressed. To obtain a realistic size of the baryon number asymmetry in the universe, we therefore have to assume that resonant enhancement of the CP asymmetries takes place, and that three degenerate right-handed neutrino masses of $O(10)$ TeV are present.

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1 Introduction

The baryon asymmetry in the universe brings cosmology and particle physics together [1]. A theoretically attractive idea [2] to produce baryon asymmetry is to apply a non-perturbative conversion mechanism of lepton asymmetry to baryon asymmetry, which exists as the sphaleron process in the standard model (SM) [3,4]. For this idea to work, a sufficient amount of $B-L$ has to be generated [5] at temperatures T between 100 and 10^{12} GeV [6,7]. With the experimental fact that the neutrinos are massive [8–11], it is plausible to believe that they are Majorana particles and hence the lepton number conservation is violated. This situation is nicely realized in the see-saw mechanism [12], which, after spontaneous symmetry breaking of $SU(2)_{\mathrm{L}} \times U(1)_Y$, generates the Majorana masses of the left-handed neutrinos in the presence of heavy right-handed neutrinos.

With the see-saw mechanism at hand, it becomes indeed possible to explain the observed ratio of baryons to photons $\eta_B = (6.2\text{–}6.9) \times 10^{-10}$ [13] by leptogenesis [14–58]. However, the introduction of the right-handed neutrinos into the SM introduces additional ambiguities in the Yukawa sector. Because of these ambiguities, the theoretical value of η_B depends on many independent parameters, so that it would be very difficult to make quantitative tests of the different mechanisms involved to produce baryon asymmetry. The origin of these ambiguities in the Yukawa sector in the SM is the missing of a more strict theoretical guide to construct the Yukawa sector.

A natural guidance to constrain the Yukawa sector is a flavor symmetry. Although there are attractive continuous symmetries, we would like to consider discrete symmetries, especially non-abelian discrete symmetries[1]. However, experimental data require that within the framework of the SM any non-abelian flavor symmetry has to be explicitly broken at low energy by operators with canonical dimensions equal to or greater than four. If the Higgs sector of the SM is so extended that a certain set of Higgs fields belong to a non-trivial representation of a non-abelian flavor group, phenomenologically viable possibilities may arise[2]. The smallest non-abelian discrete group is S_3. In this paper, we would like to consider a minimal S_3 invariant extension of the SM [69,70], in which S_3 is only softly broken at low energy, and consequently, the Yukawa sector is much more constrained than in the SM. In Sect. 2 we define the model, while investigating the independent phases in the leptonic sector. We discuss neutrino mixing and CP phase in Sect. 3, and express the average neutrino mass $\langle m_{ee} \rangle$ appearing in neutrinoless double β decay as a function of the independent phase ϕ_ν. Leptogenesis and baryon asymmetry are considered in Sect. 4, while Sect. 5 is devoted to summarizing our findings.

2 S_3 invariant extension of the standard model

2.1 Leptonic sector

We assume that the three generations of quarks and leptons belong to the reducible representation of S_3 $3 = 1 + 2$. We

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[1] Earlier papers on permutation symmetries are [59–66] for instance. See [67] for a review.

[2] Recently, phenomenologically viable models based on non-abelian discrete flavor symmetries S_3 [68–73], D_4 [74,75], A_4 [76–79], Q_4 [80] and Q_6 [81], and also on a product of abelian discrete symmetries [82–84], have been constructed. See also [85–92].

also introduce an S_3 doublet Higgs fields, $H_i (i = 1, 2)$, as well as an S_3 singlet Higgs, H_S. The S_3 invariant Yukawa interactions in the leptonic sector is given by [69][3]

$$\begin{aligned}\mathcal{L}_Y = &-y_1 \overline{L}_i H_S E_{i\mathrm{R}} - y_3 \overline{L}_S H_S E_{S\mathrm{R}} - y_2 f_{ijk} \overline{L}_i H_j E_{k\mathrm{R}} \\ &-y_4 \overline{L}_S H_i E_{i\mathrm{R}} - y_5 \overline{L}_i H_i E_{S\mathrm{R}} \\ &-h_1 \overline{L}_I \epsilon H_S^* \nu_{i\mathrm{R}} - h_3 \overline{L}_S \epsilon H_S^* \nu_{S\mathrm{R}} - h_2 f_{ijk} \overline{L}_i \epsilon H_j^* \nu_{k\mathrm{R}} \\ &-h_4 \overline{L}_S \epsilon H_i^* \nu_{i\mathrm{R}} - h_5 \overline{L}_i \epsilon H_i^* \nu_{S\mathrm{R}} \qquad (1) \\ &-\frac{1}{2} M_1 \overline{\nu}_{i\mathrm{R}} \nu_{i\mathrm{R}} - \frac{1}{2} M_S \overline{\nu}_{S\mathrm{R}} \nu_{S\mathrm{R}} + \mathrm{h.c.},\ i, j, k = 1, 2,\end{aligned}$$

where

$$\begin{aligned} f_{121} &= f_{211} = f_{112} = -f_{222} = 1, \\ f_{111} &= f_{221} = f_{122} = f_{212} = 0. \end{aligned} \qquad (2)$$

Here $L, E_\mathrm{R}, \nu_\mathrm{R}$ and H stand for left-handed charged lepton $SU(2)_\mathrm{L}$ doublets, right-handed charged leptons, right-handed neutrinos and Higgs $SU(2)_\mathrm{L}$ doublets, respectively. $\mathcal{L}_Y$ is the most general renormalizable form that is S_3 invariant. In [69] an additional abelian discrete symmetry Z_2 has been introduced to achieve a further simplification of the leptonic sector. The Z_2 parity assignment is

$$+ \quad \text{for} \quad H_i, L_S, L_i, E_{i\mathrm{R}}, E_{S\mathrm{R}}, \nu_{i\mathrm{R}}$$

and

$$- \quad \text{for} \quad H_S, \nu_{S\mathrm{R}}. \qquad (3)$$

This Z_2 forces the following Yukawa couplings to vanish[4]:

$$y_1, y_3, h_1 \quad \text{and} \quad h_5. \qquad (4)$$

Let us next figure out the structure of the CP phases. To this end, we introduce phases explicitly as follows:

$$\begin{aligned} y_a &\to \mathrm{e}^{\mathrm{i}p_{y_a}} y_a \ (a = 2, 4, 5), \\ h_a &\to \mathrm{e}^{\mathrm{i}p_{h_a}} h_a \ (a = 2, 4, 3) \end{aligned} \qquad (5)$$

for the Yukawa couplings, where the y's and h's on the right-hand side are assumed to be real and for the p's $-\pi/2 \le p \le \pi/2$, and similarly for the fields

$$\begin{aligned} &L_i \to \mathrm{e}^{\mathrm{i}p_\mathrm{L}} L_i,\ L_S \to \mathrm{e}^{\mathrm{i}p_{L_S}} L_S, \\ &E_{i\mathrm{R}} \to \mathrm{e}^{\mathrm{i}p_E} E_{i\mathrm{R}},\ E_{S\mathrm{R}} \to \mathrm{e}^{\mathrm{i}p_{E_S}} E_{S\mathrm{R}}, \\ &\nu_{i\mathrm{R}} \to \mathrm{e}^{\mathrm{i}p_\nu} \nu_{i\mathrm{R}},\ \nu_{S\mathrm{R}} \to \mathrm{e}^{\mathrm{i}p_{\nu_S}} \nu_{S\mathrm{R}}. \end{aligned} \qquad (6)$$

The phases of the right-handed neutrinos are used to absorb the phase of their Majorana masses M_1 and M_S. The phases of y_2, y_4 and y_5 can be rotated away if

$$p_\mathrm{L} = p_{y_2} + p_E,\ p_{L_S} = p_{y_4} + p_E,$$

$$p_{E_S} = -p_{y_5} + p_{y_2} + p_E \qquad (7)$$

are satisfied. So, only one free phase is left, which we assume to be p_L. Then we cancel the phase of h_2 by p_L, which implies

$$p_\mathrm{L} = p_{h_2}. \qquad (8)$$

No further phase rotation is possible, so that h_3 and h_4 can be complex in general. (Since for the mass matrices one can rotate the left-handed neutrinos and left-handed charged leptons separately, one can eliminate one more phase so that the neutrino mass matrix has only one independent phase.) However, as we will see in the following discussions that only the phase difference $p_{h_3} - p_{h_4}$ enters into the mass matrix of the left-handed neutrinos and into the CP asymmetries responsible for leptogenesis.

2.2 Higgs sector

Before we come to discuss the double beta decay of the present model, we would like to briefly summarize the feature of the Higgs sector. The present model contains five neutral physical Higgs fields; two scalars and three pseudo scalars. Their couplings to the fermions are basically fixed [69], but the Yukawa sector does not satisfy the general conditions [93,94] to suppress the tree level FCNCs. The only way to suppress the tree level FCNCs in the model is to make the Higgs particles sufficiently heavy $\gtrsim 10\,\mathrm{TeV}$, which mediate the tree level FCNCs. So, it is important to study the Higgs potential. The S_3 invariant Higgs potential V_H has been studied in [59,95]. It has turned out that all the Higgs masses obtained from V_H are proportional to VEVs, so that unless one discards the triviality constraint, they can be at most of the order of several hundreds GeV. These values are too small to suppress FCNCs [59,65].

One of the ways out of the problem is to break the S_3 symmetry softly; as soft as possible to preserve the prediction from $S_3 \times Z_2$ in the Yukawa sector. It has been observed that if the Higgs potential V_H respects S_3 as well as Z_2 invariance (Z_2 is defined in (3)), it has an additional abelian discrete symmetry S_2',

$$H_1 \leftrightarrow H_2, \qquad (9)$$

which is not a subgroup of the original S_3. Therefore, we assume that the soft breaking mass term $\hat{V}_\mathrm{SB}$ also respects this discrete symmetry S_2', while breaking $S_3 \times Z_2$ softly. The most general form is

$$\begin{aligned} \hat{V}_\mathrm{SB} = &-\mu_\mathrm{SB1}^2 (H_1^\dagger H_2 + \mathrm{h.c.}) \\ &-[\mu_\mathrm{SB2}^2\ H_S^\dagger (H_1 + H_2) + \mathrm{h.c.}\ ], \end{aligned} \qquad (10)$$

where μ_SB1 is real, but μ_SB2 can be complex. It has been shown in [95], under the assumption that μ_SB2 is real and $\langle H_S \rangle \neq 0$, that for the $S_3 \times Z_2 \times S_2'$ invariant Higgs potential with (10), only S_2' invariant VEVs

$$\langle H_S \rangle \neq 0,\ \langle H_1 \rangle = \langle H_2 \rangle \neq 0, \qquad (11)$$

[3] In [69] it is incorrectly stated that there is no CP phase in the present model.

[4] This symmetry is broken in the quark Yukawa sector. Therefore, there will be radiative corrections coming from that sector. However, they appear first at the two-loop level, and so one may assume that they are small.

can satisfy the condition that all the physical Higgs bosons, except one neutral physical Higgs boson, can become heavy $\gtrsim 10\,\mathrm{TeV}$ without having a problem with triviality. We find that even the reality assumption on μ_{SB2} can be suppressed to satisfy this condition. We would like to emphasize that the S_2' invariant VEVs (11) are the most economic VEVs in the sense that the freedom of VEVs can be completely absorbed into the Yukawa couplings so that we can derive the most general form for the fermion mass matrices

$$M = \begin{pmatrix} m_1+m_2 & m_2 & m_5 \\ m_2 & m_1-m_2 & m_5 \\ m_4 & m_4 & m_3 \end{pmatrix} \qquad (12)$$

without referring to the details of the Higgs potential.

To diagonalize the Higgs fields, we redefine the Higgs fields as

$$H_\pm = \frac{1}{\sqrt{2}}(H_1 \pm H_2), \qquad (13)$$

$$H_{\mathrm{L}} = \cos\gamma H_S + \sin\gamma H_+,\ H_H = -\sin\gamma H_S + \cos\gamma H_+$$

and

$$H_- = \begin{pmatrix} h_- \\ \frac{1}{\sqrt{2}}(h_-^0 + \mathrm{i}\chi_-) \end{pmatrix},$$

$$H_{\mathrm{L}} = \begin{pmatrix} h_{\mathrm{L}} \\ \frac{1}{\sqrt{2}}(v + h_{\mathrm{L}}^0 + \mathrm{i}\chi_{\mathrm{L}}) \end{pmatrix}, \qquad (14)$$

$$H_H = \begin{pmatrix} h_H \\ \frac{1}{\sqrt{2}}(h_H^0 + \mathrm{i}\chi_H) \end{pmatrix}, \qquad (15)$$

where

$$v_+ = \langle h_+^0\rangle,\ v_S = \langle h_S^0\rangle,$$
$$v = (v_+^2 + v_S^2)^{1/2} = 246\,\mathrm{GeV},$$
$$\sin\gamma = v_+/v,\ \cos\gamma = v_S/v. \qquad (16)$$

As we see from (14), only H_{L} has VEV, and therefore, one can identify H_{L} as the SM Higgs doublet. In fact, h_{L} and χ_{L} are the would-be Goldstone bosons. However, the neutral Higgs h_{L}^0 is not a mass eigenstate; it mixes with h_H^0. The mixing is of $O(v^2/\mu_{\mathrm{SB}}^2)$ which is at most $\sim 10^{-3}$. It is possible to kill this mixing by fine tuning of the couplings in the Higgs potential. In the following we assume this. Under this assumption, all the Higgs fields defined in (14) and (15) are mass eigenstates. In Table 1, their masses are given under the assumption that $\mu_{\mathrm{SB1}}^2, \mu_{\mathrm{SB2}}^2 \gg v^2$.

3 Neutrino mixing and neutrinoless double β decay

The fermion masses are generated from the S_2' invariant VEVs (16). Because of the Z_2 symmetry (3), the mass matrix for the charged leptons becomes

$$M_e = \begin{pmatrix} m_2 & m_2 & m_5 \\ m_2 & -m_2 & m_5 \\ m_4 & m_4 & 0 \end{pmatrix}, \qquad (17)$$

Table 1. Mass of the Higgs particles. $m_{h_{-,H}}$ should be larger than $\sim 10\,\mathrm{TeV}$ to sufficiently suppress the tree level FCNCs

Higgs	mass
h_-	$m_{h_-}^2 \simeq 2\mu_{\mathrm{SB1}}^2 + \sqrt{2}\mu_{\mathrm{SB2}}^2 \cot\gamma$
h_-^0	$m_{h_-^0}^2 \simeq m_{h_-}^2$
χ_-	$m_{\chi_-}^2 \simeq m_{h_-}^2$
h_{L}	Would-be Goldstone
h_{L}^0	$m_{h_{\mathrm{L}}}^2 = O(v^2)$
χ_{L}	Would-be Goldstone
h_H	$m_{h_H}^2 \simeq 2\sqrt{2}\mu_{\mathrm{SB2}}^2/\sin 2\gamma$
h_H^0	$m_{h_H^0}^2 \simeq m_{h_H}^2$
χ_H	$m_{\chi_H}^2 \simeq m_{h_H}^2$

where

$$m_2 = v y_2 \sin\gamma/\sqrt{2},\quad m_4 = v y_4 \sin\gamma/\sqrt{2},$$
$$m_5 = v y_5 \sin\gamma/\sqrt{2}. \qquad (18)$$

As discussed previously, the phase of all the non-vanishing Yukawa couplings y_2, y_4 and y_5 can be rotated away. So, all the mass parameters appearing in (17) are real. Diagonalization of the mass matrices is straightforward. The mass eigen values are approximately given by

$$m_e^2 = \frac{(m_4 m_5)^2}{(m_2)^2 + (m_5)^2} + O((m_4)^4), \qquad (19)$$

$$m_\mu^2 = 2(m_2)^2 + (m_4)^2 + O((m_4)^4), \qquad (20)$$

$$m_\tau^2 = 2[\ (m_2)^2 + (m_5)^2\ ] + \frac{(m_4 m_2)^2}{(m_2)^2 + (m_5)^2} + O((m_4)^4). \qquad (21)$$

Concrete values are given as $m_4/m_5 \simeq 0.00041$ and $m_2/m_5 \simeq 0.0596$ and $m_5 \simeq 1254\,\mathrm{MeV}$ to obtain $m_e = 0.51\,\mathrm{MeV}$, $m_\mu = 105.7\,\mathrm{MeV}$ and $m_\tau = 1777\,\mathrm{MeV}$. The diagonalizing unitary matrices (i.e., $U_{e\mathrm{L}}^{\mathrm{T}} M_e U_{e\mathrm{R}}$) assume a simple form in the $m_e \to 0$ limit, which is equivalent to the $m_4 \to 0$ limit. $U_{e\mathrm{L}}$ in this limit is

$$U_{e\mathrm{L}}^0 = \begin{pmatrix} 0 & -1/\sqrt{2} & 1/\sqrt{2} \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ 1 & 0 & 0 \end{pmatrix}. \qquad (22)$$

We shall consider this limit later on.

Similarly, the Dirac neutrino mass matrix is given by

$$M_{\mathrm{D}} = \begin{pmatrix} m_{\mathrm{D2}} & m_{\mathrm{D2}} & 0 \\ m_{\mathrm{D2}} & -m_{\mathrm{D2}} & 0 \\ m_{\mathrm{D4}} & m_{\mathrm{D4}} & m_{\mathrm{D3}} \end{pmatrix}, \qquad (23)$$

where

$$m_{\mathrm{D2}} = v h_2 \sin\gamma/\sqrt{2}, m_{\mathrm{D4}} = v h_4 \mathrm{e}^{\mathrm{i}p_{h_4}} \sin\gamma/\sqrt{2},$$
$$m_{\mathrm{D3}} = v\cos\gamma h_3 \mathrm{e}^{\mathrm{i}p_{h_3}}. \qquad (24)$$

For the mass matrices one can rotate the left-handed charged leptons and the left-handed neutrinos separately. So, we rotate ν_{SL} to absorb the phase of m_{D_4}, and we rewrite $M_{\rm D}$ as

$$M_{\rm D}=\begin{pmatrix} m_{\rm D2} & m_{\rm D2} & 0\\ m_{\rm D2} & -m_{\rm D2} & 0\\ m_{\rm D4} & m_{\rm D4} & m_{\rm D3}\exp{\rm i}\varphi_3\end{pmatrix}, \quad (25)$$

where

$$\varphi_3 = p_{h_4}-p_{h_3}\ (-\pi/2\le\varphi_3\le\pi/2), \quad (26)$$

and the Dirac mass parameters $m_{\rm D}$ in (25) are all real numbers.

The Majorana masses for $\nu_{\rm L}$ can be obtained from the see-saw mechanism, and the corresponding mass matrix is given by $M_\nu = M_{\rm D}\tilde{M}^{-1}(M_{\rm D})^{\rm T}$, where $\tilde{M} = {\rm diag}(M_1, M_1, M_S)$. We have assumed that the phases of the right-handed neutrinos are used to rotate away the phase of M_1 and M_S. So, we may assume that they are real positive numbers. To express M_ν in a simple form we rescale the Dirac neutrino masses according to

$$m_{\rm D2}\to\rho_2=m_{\rm D2}/\sqrt{M_1},\ m_{\rm D4}\to\rho_4=m_{\rm D4}/\sqrt{M_1},$$
$$m_{\rm D3}\to\rho_3=m_{\rm D3}/\sqrt{M_S}. \quad (27)$$

Thanks to the Z_2 symmetry (3), the mass matrix M_ν takes a simple form:

$$M_\nu = M_{\rm D}\tilde{M}^{-1}(M_{\rm D})^{\rm T} \quad (28)$$
$$=\begin{pmatrix} 2(\rho_2)^2 & 0 & 2\rho_2\rho_4\\ 0 & 2(\rho_2)^2 & 0\\ 2\rho_2\rho_4 & 0 & 2(\rho_4)^2+(\rho_3)^2\exp{\rm i}2\varphi_3\end{pmatrix}.$$

The ρ's in (28) are real numbers. One can convince oneself that M_ν can be diagonalized as [70]

$$U_\nu^{\rm T}M_\nu U_\nu=\begin{pmatrix} m_{\nu_1}{\rm e}^{{\rm i}\phi_1-{\rm i}\phi_\nu} & 0 & 0\\ 0 & m_{\nu_2}{\rm e}^{{\rm i}\phi_2+{\rm i}\phi_\nu} & 0\\ 0 & 0 & m_{\nu_3}\end{pmatrix}, \quad (29)$$

where

$$U_\nu=\begin{pmatrix} -s_{12} & c_{12}{\rm e}^{{\rm i}\phi_\nu} & 0\\ 0 & 0 & 1\\ c_{12}{\rm e}^{-{\rm i}\phi_\nu} & s_{12} & 0\end{pmatrix}, \quad (30)$$

$$m_{\nu_3}\sin\phi_\nu = m_{\nu_2}\sin\phi_2 = m_{\nu_1}\sin\phi_1,$$
$$2\varphi_3=\phi_1+\phi_2 \quad (31)$$
$$m_{\nu_3}=2\rho_2^2,\ \frac{m_{\nu_1}m_{\nu_2}}{m_{\nu_3}}=\rho_3^2, \quad (32)$$
$$\tan\phi_\nu=\frac{\rho_3^2\sin 2\varphi_3}{2(\rho_2^2+\rho_4^2)+\rho_3^2\cos 2\varphi_3}, \quad (33)$$

and $c_{12}=\cos\theta_{12}$ and $s_{12}=\sin\theta_{12}$. We also find that

$$\tan^2\theta_{12}=\frac{(m_{\nu_2}^2-m_{\nu_3}^2\sin^2\phi_\nu)^{1/2}-m_{\nu_3}|\cos\phi_\nu|}{(m_{\nu_1}^2-m_{\nu_3}^2\sin^2\phi_\nu)^{1/2}+m_{\nu_3}|\cos\phi_\nu|}, \quad (34)$$

from which we find

$$\frac{m_{\nu_2}^2}{\Delta m_{23}^2}=\frac{(1+2t_{12}^2+t_{12}^4-rt_{12}^4)^2}{4t_{12}^2(1+t_{12}^2)(1+t_{12}^2-rt_{12}^2)\cos^2\phi_\nu}-\tan^2\phi_\nu \quad (35)$$
$$\simeq\frac{1}{\sin^2 2\theta_{12}\cos^2\phi_\nu}-\tan^2\phi_\nu \quad \text{for} \quad |r|\ll 1, \quad (36)$$

where $t_{12}=\tan\theta_{12}, r=\Delta m_{21}^2/\Delta m_{23}^2$. It can also be shown that only an inverted mass spectrum

$$m_{\nu_3}<m_{\nu_1},m_{\nu_2} \quad (37)$$

is consistent with the experimental constraint $|\Delta m_{21}^2|<|\Delta m_{23}^2|$ in the present model. Note that (32) is satisfied for

$$2\varphi_3=\phi_1+\phi_2\sim\pm\pi \quad (38)$$

and *not* for $\phi_1\sim\phi_2$. That is, if $2\varphi_3\sim+(-)\pi$, then $\cos\phi_1<(>)0$ and $\cos\phi_2>(<)0$. Now the product $U_{e{\rm L}}^\dagger P U_\nu$ defines a neutrino mixing matrix $V_{\rm MNS}$, where

$$P=\begin{pmatrix}1&0&0\\0&1&0\\0&0&{\rm e}^{{\rm i}p'_{h_4}}\end{pmatrix},\quad p'_{h_4}=p_{h_4}-p_{h_2}+p_{y_2}-p_{y_4}.$$

For our purpose it is sufficient to use the approximate unitary matrix $U_{e{\rm L}}^0$ given in (22) which is obtained in the limit that the electron mass is zero. We denote the approximate neutrino mixing matrix by $V_{\rm MNS}^0$ obtained from $U_{e{\rm L}}^0$ and U_ν. The product $U_{e{\rm L}}^{0\dagger}PU_\nu$ can be brought by an appropriate phase transformation to a popular form:

$$V_{\rm MNS}\simeq V_{\rm MNS}^0=$$
$$\begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}{\rm e}^{-{\rm i}\delta}\\ -s_{12}c_{23}-c_{12}s_{23}s_{13}{\rm e}^{{\rm i}\delta} & c_{12}c_{23}-s_{12}s_{23}s_{13}{\rm e}^{{\rm i}\delta} & s_{23}c_{13}\\ s_{12}s_{23}-c_{12}c_{23}s_{13}{\rm e}^{{\rm i}\delta} & -c_{12}s_{23}-s_{12}c_{23}s_{13}{\rm e}^{{\rm i}\delta} & c_{23}c_{13}\end{pmatrix}$$
$$\times\begin{pmatrix}1&0&0\\0&{\rm e}^{{\rm i}\alpha}&0\\0&0&{\rm e}^{{\rm i}\,\beta}\end{pmatrix}. \quad (39)$$

with

$$s_{13}=0,\ t_{23}=\frac{s_{23}}{c_{23}}=1, \quad (40)$$

$$\sin 2\alpha=\sin(\phi_1-\phi_2)$$
$$=\pm\frac{m_{\nu_3}\sin\phi_\nu}{m_{\nu_1}m_{\nu_2}} \quad (41)$$
$$\times\left(\sqrt{m_{\nu_2}^2-m_{\nu_3}^2\sin^2\phi_\nu}+\sqrt{m_{\nu_1}^2-m_{\nu_3}^2\sin^2\phi_\nu}\right)$$
$$\simeq\pm 2\sin\phi_\nu(m_{\nu_3}/m_{\nu_2})\sqrt{1-(m_{\nu_3}/m_{\nu_2})^2\sin^2\phi_\nu},$$

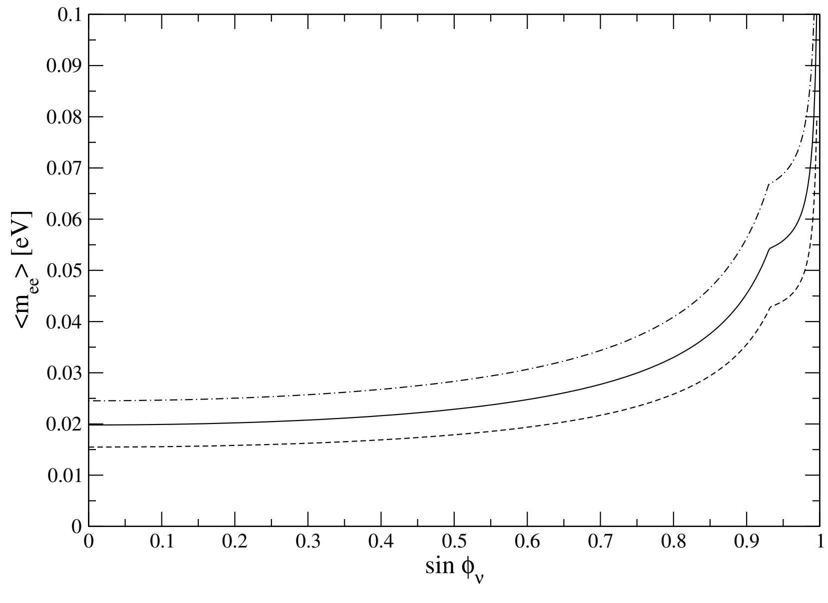

Fig. 1. The effective Majorana mass $\langle m_{ee}\rangle$ as a function of $\sin\phi_\nu$ with $\sin^2\theta_{12} = 0.3$ and $\Delta m^2_{21} = 6.9\times10^{-5}$ eV^2. The dashed, solid and dot-dashed lines stand for $\Delta m^2_{23} = 1.4, 2.3$ and 3.0×10^{-3} eV^2, respectively. The Δm^2_{21} dependence is very small

$$\begin{aligned}\sin 2\beta &= \sin(\phi_1-\phi_\nu)\\ &= \pm\frac{\sin\phi_\nu}{m_{\nu_1}}\\ &\quad\times\left(m_{\nu_3}\sqrt{1-\sin^2\phi_\nu}+\sqrt{m^2_{\nu_1}-m^2_{\nu_3}\sin^2\phi_\nu}\right)\end{aligned}\tag{42}$$

for $2\varphi_2\sim\pm\pi$, where ϕ_1,ϕ_2 and ϕ_ν are defined in (32)[5].

The effective Majorana mass $\langle m_{ee}\rangle$ in neutrinoless double β decay is given by

$$\langle m_{ee}\rangle = \left|\sum_{i=1}^{3} m_{\nu_i}V^2_{ei}\right| \simeq |m_{\nu_1}c^2_{12}+m_{\nu_2}s^2_{12}\exp \mathrm{i}2\alpha|,\tag{43}$$

where ϕ_ν and α are given in (32) and (41), respectively. In Fig. 1 we plot $\langle m_{ee}\rangle$ as a function of $\sin\phi_\nu$ for $\sin^2\theta_{12} = 0.3, \Delta m^2_{21} = 6.9\times10^{-5}\,\mathrm{eV}^2$ and $\Delta m^2_{23} = 1.4, 2.3, 3.0\times10^{-3}$ eV^2 [97]. As we can see from Fig. 1, the effective Majorana mass stays at about its minimal value $\langle m_{ee}\rangle_{\min}$ for a wide range of $\sin\phi_\nu$. Since $\langle m_{ee}\rangle_{\min}$ is approximately equal to $\sqrt{\Delta m^2_{23}}/\sin 2\theta_{12} = (0.034\text{–}0.069)\,\mathrm{eV}$, it is consistent with recent experiments [13,98] and is within an accessible range of future experiments [99].

Noticing that (32), (38) and (41), one obtains[6] (for $2\varphi_3\sim-\pi$)

$$\begin{aligned}\sin 2\varphi_3 &= -\frac{m_{\nu_3}}{m_{\nu_1}}\sin\phi_\nu\left[1-\left(\frac{m_{\nu_3}}{m_{\nu_2}}\sin\phi_\nu\right)^2\right]^{1/2}\\ &\quad+\frac{m_{\nu_3}}{m_{\nu_2}}\sin\phi_\nu\left[1-\left(\frac{m_{\nu_3}}{m_{\nu_1}}\sin\phi_\nu\right)^2\right]^{1/2}\\ &\simeq -\frac{m_{\nu_3}}{2m^3_{\nu_2}}\frac{\Delta m^2_{21}\sin\phi_\nu}{(1-(m_{\nu_3}/m_{\nu_2})^2\sin^2\phi_\nu)^{1/2}},\end{aligned}\tag{44}$$

where $\Delta m^2_{21}/m^2_{\nu_2}\ll 1$ is assumed. As one sees from (35), (37), (38), (41) and (42), once $\theta_{12}, \Delta m^2_{21}$ and Δm^2_{23} are given, the only free parameter is ϕ_ν. Numerically one finds

$$\sin 2\varphi_3 \simeq -(0.0034\text{–}0.013)\sin\phi_\nu,\tag{45}$$

where we have used $\sin^2\theta_{12} = 0.3, 1.4\,\mathrm{eV}^2 \lesssim \Delta m^2_{21}\times 10^5 \lesssim 3.0\,\mathrm{eV}^2$ and $6.1\,\mathrm{eV}^2 \lesssim \Delta m^2_{23}\times10^3 \lesssim 8.4\,\mathrm{eV}^2$ [97]. Therefore, CP asymmetry, being proportional to $\sin 2\varphi_3$, is very small in the present model, even if the CP phase appearing in neutrinoless double β decays is large[7].

4 Leptogenesis

4.1 CP phase

Before we start to compute CP asymmetries, we first would like to consider the mixing of the charged leptons in the limit that the mass of the electron vanishes. That is, we approximate the unitary matrix which defines the mass eigenstates of the charged leptons by $U^0_{e\mathrm{L}}$; see (22). We next rewrite the Yukawa interactions (1) in terms of the mass eigenstates for the Higgses, (14) and (15), and the charged leptons. The relevant part for leptogenesis becomes

$$\begin{aligned}\mathcal{L}_h &= -h^-_{IJ}\overline{\hat{L}}_I\epsilon H^*_-\nu_{J\mathrm{R}} - h^H_{IJ}\overline{\hat{L}}_I\epsilon H^*_H\nu_{J\mathrm{R}}\\ &\quad -h^L_{IJ}\overline{\hat{L}}_I\epsilon H^*_{\mathrm{L}}\nu_{J\mathrm{R}}\end{aligned}\tag{46}$$

with

$$\hat{L} = U^0_{e\mathrm{L}}L = \left(\begin{pmatrix}\nu_{e\mathrm{L}}\\ e_{\mathrm{L}}\end{pmatrix}, \begin{pmatrix}\nu_{\mu\mathrm{L}}\\ \mu_{\mathrm{L}}\end{pmatrix}, \begin{pmatrix}\nu_{\tau\mathrm{L}}\\ \tau_{\mathrm{L}}\end{pmatrix}\right),\tag{47}$$

[5] For a non-vanishing electron mass, we have $s_{13}\simeq m_e/\sqrt{2}m_\mu \simeq 0.0034$ and $\delta = p_{h_4}-\phi_\nu$. Unfortunately, this value of s_{13} is too small to be measured [96].

[6] As we will see in the next section, the ratio of baryons to photons η_B is proportional to $-\sin 2\varphi_3$.

[7] Models in which the CP phases in the neutrino mixing matrix are closely related to those for leptogenesis have been considered, for instance, in [42–46].

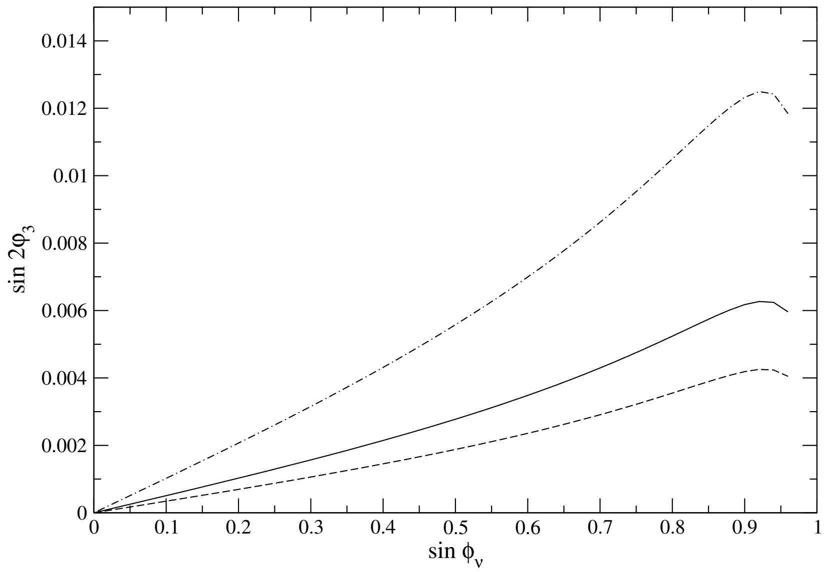

Fig. 2. $\sin 2\varphi_3$ versus $\sin\phi_\nu$ with $\sin^2\theta_{12} = 0.3$ in the case of $2\varphi_3 \sim +\pi$. The case of $2\varphi_3 \sim -\pi$ is the same except for the sign of $\sin 2\varphi_3$. The dot-dashed, solid and dashed lines stand for $(\Delta m^2_{23}\times 10^3, \Delta m^2_{21}\times 10^5) = (3.0, 6.1), (2.3, 6.9)$ and $(1.4, 8.4)$ eV^2, respectively

where

$$h^H_{IJ} = \begin{pmatrix} \cos\gamma h_4 e^{ip_{h_4}}/\sqrt{2} & \cos\gamma h_4 e^{ip_{h_4}}/\sqrt{2} & -\sin\gamma h_3 e^{ip_{h_3}} \\ 0 & -\cos\gamma h_2 & 0 \\ \cos\gamma h_2 & 0 & 0 \end{pmatrix}, \tag{48}$$

$$h^L_{IJ} = \begin{pmatrix} \sin\gamma h_4 e^{ip_{h_4}}/\sqrt{2} & \sin\gamma h_4 e^{ip_{h_4}}/\sqrt{2} & \cos\gamma h_3 e^{ip_{h_3}} \\ 0 & -\sin\gamma h_2 & 0 \\ \sin\gamma h_2 & 0 & 0 \end{pmatrix}, \tag{49}$$

$$h^-_{IJ} = \begin{pmatrix} h_4 e^{ip_{h_4}}/\sqrt{2} & -h_4 e^{ip_{h_4}}/\sqrt{2} & 0 \\ h_2 & 0 & 0 \\ 0 & h_2 & 0 \end{pmatrix}, \tag{50}$$

where γ is defined in (16).

There are two types of diagrams that contribute to CP asymmetries, vertex diagrams [2, 14] and self-energy diagrams [17,18]. Non-vanishing CP asymmetries are proportional to the imaginary part of $(h_{KI}h^*_{KJ})^2$. In the present case, there are three Higgs fields, and one finds that if one neglects the mass difference of the Higgs bosons, the vertex correction is proportional to the imaginary part of

$$\sum_{J,K,M}\ \sum_{A,B=H,L,-} (h^A_{JI}h^{B*}_{JK})(h^B_{MI}h^{A*}_{MK}), \tag{51}$$

while the self-energy correction is proportional to the imaginary part of

$$\sum_{J,K,M}\ \sum_{A,B=H,L,-} (h^A_{JI}h^{A*}_{JK})(h^B_{MI}h^{B*}_{MK}). \tag{52}$$

Since h_{I1} and h_{I2} have the same phase, only the case $I = 1, 2, K = 3$ or $I = 3, K = 1, 2$ yields non-vanishing CP asymmetries. Therefore, the matrix h^-_{IJ} cannot contribute to CP asymmetries. Moreover, since $h^{H,L}_{i3} = 0$ $(i = 2, 3)$, both the vertex and self-energy contributions become proportional to the imaginary part of

$$\sum_{A,B=H,L} (h^A_{1i}h^{A*}_{13})(h^B_{1i}h^{B*}_{13}), i = 1, 2. \tag{53}$$

However, from (48) and (49) we obtain

$$h^H_{1i}h^{H*}_{13} + h^L_{iI}h^{L*}_{13} = 0, \tag{54}$$

implying that CP asymmetries vanish, if the mass differences of the Higgs bosons are neglected. That is, CP asymmetries, generated in one-loop with H_L and H_H exchanges, cancel with each other. In the presence of the soft $S_3 \times Z_2$ breaking mass terms (10), the mass of H_H can be considerably different from that of H_L, as we can see from Table 1. Consequently, there will be non-vanishing CP asymmetries in a realistic case, in which H_- and H_H are made heavy by the soft $S_3 \times Z_2$ breaking terms to suppress the tree level FCNCs.

After so many discussions about the S_3 limit, we find that

$$\begin{aligned} \mathrm{Im}\,[h^{H,L}_{1I}h^{H,L*}_{1J}]^2 &\sim (h_3h_4)^2 \sin 2(p_{h_3} - p_{h_4}) \\ &= -(h_3h_4)^2 \sin 2\varphi_3, \end{aligned} \tag{55}$$

with $I = 1, 2,\ J = 3$ or $I = 3,\ J = 1, 2$, where φ_3 is given in (26), which is the only phase left over in the neutrino mass matrix. Therefore, the CP phase appearing in the neutrino mixing is the same as that for CP asymmetries.

4.2 CP asymmetries and baryon number asymmetry

We first calculate the total decay width of the right-handed neutrinos. The relevant parts of the Yukawa interactions for this purpose are given in (46)–(50). For the S_3 singlet right-handed neutrino $\nu_{S\mathrm{R}}$ one finds

$$
\begin{aligned}
&\Gamma_{ST}\\
&= \Gamma_S[l+H_{\mathrm{L}}]+\Gamma_S[l+H_H]+\Gamma_S[l^c+H_{\mathrm{L}}^c]\\
&\quad +\Gamma_S[l^c+H_H^c]\\
&= \frac{1}{8\pi}h_3^2 M_S \qquad (56)\\
&\quad \times\left[\cos^2\gamma+\sin^2\gamma\left[\left(1-\frac{m_{h_H}^2}{M_S^2}\right)^2\theta(M_S-m_{h_H})\right]\right]\\
&\to \frac{1}{8\pi}h_3^2 M_S \quad \text{as} \quad m_{h_H}/M_S\to 0,\\
&\to \frac{1}{8\pi}h_3^2 M_S\cos^2\gamma=\frac{1}{8\pi}\left(\frac{m_{\nu_1}m_{\nu_2}}{m_{\nu_3}}\right)\left(\frac{M_S}{v^2}\right)^2\\
&\qquad \text{as} \quad m_{h_H}/M_S\to 1,
\end{aligned}
$$

where the first term results from the decay into the SM Higgs H_{L}, and the second term comes from the decay into H_H with mass m_{h_H}. The last equality follows from (24), (25), (27) and (32). Similarly, one finds the total decay width of $\nu_{1\mathrm{R}}$ and $\nu_{2\mathrm{R}}$

$$
\begin{aligned}
&\Gamma_{1T}=\Gamma_{2T}\\
&= \frac{1}{8\pi}\left(\frac{1}{2}h_4^2+h_2^2\right)M_1\\
&\quad\times\left[\sin^2\gamma+\cos^2\gamma\left[\left(1-\frac{m_{h_H}^2}{M_1^2}\right)^2\theta(M_1-m_{h_H})\right]\right.\\
&\quad\left.+\left[\left(1-\frac{m_{h_-}^2}{M_1^2}\right)^2\theta(M_1-m_{h_-})\right]\right] \qquad (57)\\
&\to \frac{1}{4\pi}\left(\frac{1}{2}h_4^2+h_2^2\right)M_1 \quad \text{as} \quad m_{h_H}/M_1, m_{h_-}/M_1\to 0,\\
&\to \frac{1}{8\pi}\left(\frac{1}{2}h_4^2+h_2^2\right)M_1\sin^2\gamma\\
&= \frac{1}{8\pi}(m_{\nu_3}+\rho_4^2)\left(\frac{M_1}{v}\right)^2\\
&\qquad \text{as} \quad m_{h_H}/M_1, m_{h_-}/M_1\to 1,
\end{aligned}
$$

where $M_1=M_2$ is assumed.

Because of (45), i.e. $|\sin\varphi_3|\lesssim 0.013$, one needs an enhancement to obtain a realistic value of baryon asymmetry. A nice way is the resonant enhancement [17,18,22], which we consider below[8]. Since in this case the self-energy contributions to CP asymmetries dominate, we consider only them and neglect the contributions coming from the vertex diagrams. In the S_3 symmetric limit, M_1 is equal to M_2. This relation is modified to $M_1=M_2+O(m_\nu)$ because of spontaneous symmetry breaking of S_3. So, there is a natural degeneracy of $\nu_{1\mathrm{R}}$ and $\nu_{2\mathrm{R}}$. However, there is no resonant enhancement between $\nu_{1\mathrm{R}}$ and $\nu_{2\mathrm{R}}$, because $\mathrm{Im}h_{i1}^{H,L}(h_{i2}^{H,L})^*=0$ as we can see from (48) and (49). Therefore, we may neglect this small correction, and we have to assume that $M_1=M_2\simeq M_S$. Introducing the notation

$$
\Delta M^2/M_S^2=1-\frac{M_1^2}{M_S^2}=1-x\sim\ 0, \qquad (58)
$$

we find that

$$
\begin{aligned}
\Delta\Gamma_S&=\Gamma_S[l+H_{\mathrm{L}}]+\Gamma_S[l+H_H]-\Gamma_S[l^c+H_{\mathrm{L}}^c]\\
&\quad-\Gamma_S[l^c+H_H^c]\\
&=\frac{1}{64\pi^2}(h_4h_3)^2(\sin^2\gamma\cos^2\gamma)\sin 2\varphi_3\\
&\quad\times[1-(1-y_H)^2\theta(1-y_H)]^2\frac{M_1}{1-x}, \qquad (59)
\end{aligned}
$$

and $\Delta\Gamma_1\simeq\Delta\Gamma_S/2$ for $M_S\simeq M_1$. We have assumed that

$$
\Gamma_{ST,1T}/M_S\ll|\Delta M^2/M_S^2| \qquad (60)
$$

to use the approximate formula (59). From these calculations we obtain the CP asymmetries

$$
\begin{aligned}
\epsilon_S&=\frac{\Delta\Gamma_S}{\Gamma_{ST}}\\
&=\frac{1}{8\pi}h_4^2\sin 2\varphi_3 \qquad (61)\\
&\quad\times\frac{[1-(1-y_H)^2\theta(1-y_H)]^2}{[1/\sin^2\gamma+(1-y_H)^2\theta(1-y_H)/\cos^2\gamma]}\ \frac{\sqrt{x}}{1-x},\\
\epsilon_1&=\epsilon_2\simeq\frac{1}{2}\frac{\Gamma_{ST}}{\Gamma_{1T}}\epsilon_S, \qquad (62)
\end{aligned}
$$

where

$$
y_H=\frac{m_{h_H}^2}{M_S^2}. \qquad (63)
$$

From (61) various limits may be obtained:

$$
\begin{aligned}
\epsilon_S&\to 0 \quad \text{as} \quad y_H\to 0 \qquad (64)\\
&\to\frac{1}{8\pi}h_4^2\sin 2\varphi_3\sin^2\gamma\frac{\sqrt{x}}{1-x}\\
&=\frac{1}{4\pi}\left(\frac{\rho_4^2M_S}{v^2}\right)\sin 2\varphi_3\frac{\sqrt{x}}{1-x} \quad \text{as} \quad y_H\to\ 1, \qquad (65)
\end{aligned}
$$

where we have used (24), (25) and (27).

To be definite we assume $y_H, y_-=m_{h_-}^2/M_S^2\sim 1$ in the following discussions. Then only the SM Higgs H_{L} contributes to CP asymmetries, and the phase ϕ_ν (or φ_3), $\sin^2\gamma$ (defined in (16)) and the effective mass

$$
M_{\mathrm{eff}}=\frac{M_1}{1-x}=\frac{M_S^2M_1}{M_S^2-M_1^2}\simeq\frac{M_S^3}{M_S^2-M_1^2} \qquad (66)
$$

are the only independent parameters. The lepton and baryon asymmetries $Y_L=n_L/s$ and $Y_B=n_B/s$ (n_L, n_B

[8] See, for instance, [47–56] for recent models with resonant enhancement of leptogenesis. See also [58].

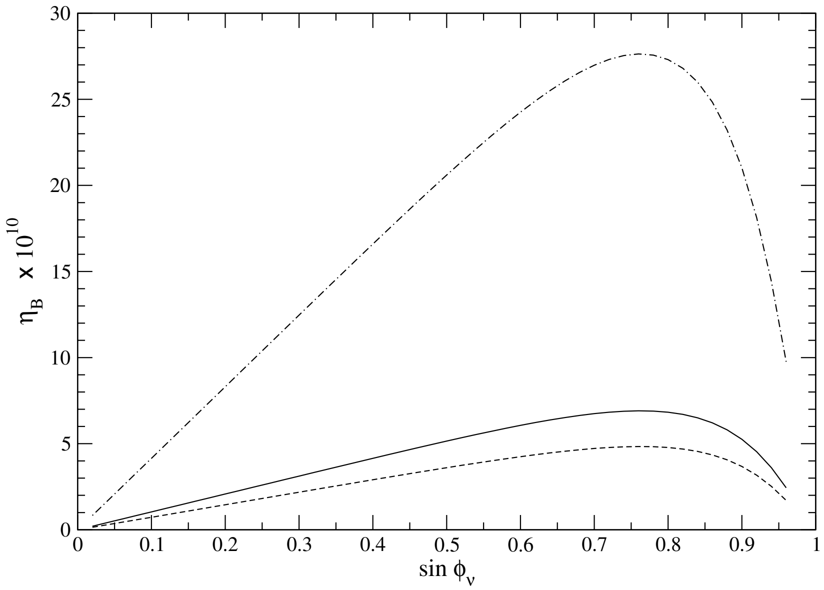

Fig. 3. η_B versus $\sin\phi_\nu$. The dot-dashed, solid and dotted lines correspond to $M_{\rm eff} = M_1/(1-x) = 4.0, 1.0, 0.7 \times 10^{13}$ GeV. We have assumed that $m_{h_H}, m_{h_-} \simeq M_S$ and used $\sin^2\theta_{12} = 0.3, \Delta m^2_{23} = 2.3 \times 10^{-3}\,\mathrm{eV}^2, \Delta m^2_{21} = 6.9 \times 10^{-5}\,\mathrm{eV}^2$. The experimental value of $\eta_B \times 10^{10}$ is about 6.5 [13]

are the lepton and baryon number density, and s is the entropy density) are given by [5]

$$Y_L \simeq \kappa_S \epsilon_S/g^* + 2\kappa_1\epsilon_1/g^* \quad \text{with} \quad g^* \simeq 120, \tag{67}$$

$$Y_B = \frac{\omega}{\omega-1} Y_L, \tag{68}$$

$$\omega = \frac{8N_F + 4N_H}{22N_F + 13N_H} \simeq 0.34 \quad \text{for} \quad N_F = 3, N_H = 3, \tag{69}$$

where $k_{S(1)}$ is the dilution factor for the CP asymmetry $\epsilon_{S(1)}$, and g^* is the effective number of degrees of freedom at the temperature $T = M_S \simeq M_1 = M_2$. We have taken into account all the degrees of freedom in g^* including three right-handed neutrinos and three Higgs doublets. The dilution factors can be approximately written as [1,22,23]

$$\kappa_S \simeq \frac{0.3}{K_S[\ln K_S]^{3/5}}, \; \kappa_1 \simeq \frac{0.3}{K_1[\ln K_1]^{3/5}}, \tag{70}$$

where

$$K_S = \Gamma_{ST}/H_{ST}$$

$$= \frac{h_3^2}{16\pi}\frac{M_{\rm PL}}{1.66\sqrt{g^*}M_S} = \frac{\rho_3^2}{8\pi v^2}\frac{M_{\rm PL}}{1.66\sqrt{g^*}} \tag{71}$$

$$\simeq 4.4 \times 10^2 \frac{(m_{\nu_1}/1\,\mathrm{eV})(m_{\nu_2}/1\,\mathrm{eV})}{(m_{\nu_3}/1\,\mathrm{eV})}, \tag{72}$$

$$K_1 = \frac{\rho_4^2 + m_{\nu_3}}{8\pi v^2}\frac{M_{\rm PL}}{1.66\sqrt{g^*}}$$

$$\simeq 4.4 \times 10^2(\rho_4^2 + m_{\nu_3})/1\,\mathrm{eV}, \tag{73}$$

where the ρ's are given in (27), and (32) is used. [The approximate formula (70) is applicable for $10 \lesssim K_{S,1} \lesssim 10^6$.] Note that using (32) and (33), we can express ρ_4^2 in terms of the neutrino masses, ϕ_ν and φ_3, and we find that the $\sin\gamma$ dependence in κ_S and κ_1 cancels so that the lepton asymmetry and hence the baryon asymmetry Y_B does not depend on $\sin\gamma$. Finally, the ratio of the baryon number density to the photon density η_B is given by

$$\eta_B \simeq 7.04 Y_B \simeq -3.0 \times 10^{-2}(\kappa_S\epsilon_S + 2\kappa_1\epsilon_1). \tag{74}$$

In Fig. 3 η_B as a function of $\sin\phi_\nu$ is plotted for three different values of $M_{\rm eff} = M_1/(1-x) = 4.0, 1.0, 0.7 \times 10^{13}$ GeV. As we see from Fig. 3, η_B becomes maximal around $\sin\phi_\nu \simeq 0.75$. To obtain a realistic value of $n_B (\simeq 6.5 \times 10^{-10})$, the effective mass $|M_{\rm eff}| = M_1/|1-x|$ should be of $O(10^{13})$ GeV, which means that $|1-x|$ has to be very small if M_1 is much smaller than 10^{13} GeV. We have to fine tune M_S so that $|1-x| = |1 - M_1^2/M_S^2| \simeq 10^{-9}$ for $M_S = 10$ TeV, for instance.

Let us at last discuss how fine this fine tuning is. A fine tuning is unnatural if radiative corrections are larger than the fine tuning. The one-loop radiative correction to the right-handed neutrino masses may be estimated to be $\delta M \sim Mh^2/16\pi^2$, where h stands for the generic Yukawa couplings in (1), and M for M_1 and M_S. So, the fine tuning of $(1-x)$ will be natural if

$$|1-x| = |1 - M_1^2/M_S^2| > \delta M/M \sim h^2/16\pi^2. \tag{75}$$

This condition, however, is weaker than the condition $|M_S^2 - M_1^2| \gg M_S\Gamma_{ST}$ and $M_1\Gamma_{1T}$, for the approximate formula (61) for the one-loop self-energy diagram to be applicable [22]. The latter condition is equivalent to

$$|1-x| \gg \frac{1}{8\pi}(h_2^2\sin^2\gamma, h_2^2\sin^2\gamma, h_3^2\cos^2\gamma), \tag{76}$$

as we can see from (56) and (57). Therefore, if this condition is satisfied, the naturalness condition is automatically satisfied. The value of h can be estimated from (24), (25), (27) and (32):

$$\frac{1}{8\pi}h_2^2\sin^2\gamma = \frac{1}{8\pi}\left(\frac{m_{\nu_3}}{v}\right)\left(\frac{M_1}{v}\right) \lesssim 10^{-13}\frac{M_1}{v}, \tag{77}$$

$$
\begin{aligned}
&\frac{1}{8\pi} h_4^2 \sin^2\gamma \\
&= \frac{1}{16\pi}\left(\frac{m_{\nu_1} m_{\nu_2}}{m_{\nu_3} v}\frac{\sin 2\varphi_3}{\tan\phi_\nu} + \frac{m_{\nu_1} m_{\nu_2}}{m_{\nu_3} v}\cos 2\varphi_3 - \frac{m_{\nu_3}}{v}\right) \\
&\quad\times\left(\frac{M_1}{v}\right) \\
&\lesssim 10^{-13}\frac{M_1}{v},
\end{aligned}
\tag{78}
$$

$$
\begin{aligned}
&\frac{1}{8\pi} h_3^2 \cos^2\gamma \\
&= \frac{1}{8\pi}\left(\frac{m_{\nu_1} m_{\nu_2}/m_{\nu_3}}{v}\right)\left(\frac{M_S}{v}\right) \lesssim 10^{-13}\frac{M_S}{v}.
\end{aligned}
\tag{79}
$$

The last inequality is obtained as follows. First we use the fact that in the present model an inverted hierarchy is predicted, and the experimental bound [13] $m_\nu \lesssim 0.2$ eV. The ratio $|\sin 2\varphi_3/\tan\phi_\nu|$ is less than 0.013 because of (45). Further from (36) we find

$$
m_{\nu_2}^2 \lesssim \frac{\Delta m_{23}^2}{\sin^2 2\theta_{12}}, \tag{80}
$$

which gives

$$
\begin{aligned}
\frac{m_{\nu_2}^2}{m_{\nu_3}^2} &= \left(1 - \frac{\Delta m_{23}^2}{m_{\nu_2}^2}\right)^{-1} \\
&\lesssim \left(1 - \sin^2 2\theta_{12}\right)^{-1} = \cos^{-2} 2\theta_{12} \lesssim (4.6)^2.
\end{aligned}
\tag{81}
$$

Therefore, the condition (76) becomes

$$
|1 - x| \gg 10^{-13}\frac{M_1}{v}. \tag{82}
$$

In terms of the effective mass M_{eff} one finally finds

$$
|M_{\text{eff}}| = \frac{M_1}{|1-x|} \ll 10^{13} v \simeq 10^{15}\,\text{GeV}. \tag{83}
$$

Therefore, the criterion on the validity of the approximate formula (61) does not depend on the mass of the right-handed neutrinos. We recall that if (83) is satisfied, the naturalness condition (75) is automatically satisfied. For $M_S = 10\,\text{TeV}$, for instance, we obtain $|1-x| \gg 4\times 10^{-13}$ which implies that the fine tuning of $|1-x| \simeq 10^{-9}$ to obtain $M_{\text{eff}} \simeq 10^{13}\,\text{GeV}$ is not unnatural, and the use of the approximate formula (61) is justified. The main reason of the independence of the right-handed neutrinos masses, M_1 and M_S, is the see-saw mechanism; the smaller M_1 and M_S are, the finer fine tuning of $(1-x)$ is allowed because of (76).

As we can see from (64), the CP asymmetries in the present model vanish if the Higgs masses are much smaller than the right-handed neutrinos masses. On one hand, the heavier the Higgs masses are, the finer fine tuning is needed in the Higgs sector. The constraints coming from FCNCs, on the other hand, require them to be larger than $O(10)$ TeV [59, 65]. Therefore, if we would like to explain the observed baryon asymmetry from leptogenesis within the framework of the present model, it is theoretically desirable to have right-handed neutrinos masses less than, say, $O(100)$ TeV.

5 Conclusion

In this paper we considered a minimal S_3 extension of the SM and investigated the possibility to explain the observed baryon asymmetry in the universe through leptogenesis [2]. Below we would like to summarize our findings.

(1) As in [69, 70], we assumed an additional discrete symmetry (3) to increase the predictive power in the leptonic sector. The leptonic sector of the Yukawa interactions contains two independent phases p_{h_4} and p_{h_3}. We found that in the limit that the electron mass vanishes, only the combination $2\varphi_3 = 2(p_{h_3} - p_{h_4})$ enters into the neutrino mixing matrix V_{MNS} as well as into the CP asymmetries ϵ's responsible for leptogenesis. (In this limit, V_{e3} vanishes.) However, because of (45), we obtained $\delta_{CP} = |\sin 2\varphi_3| \lesssim 0.013$.

(2) It turned out that in the S_3 symmetric limit, the CP asymmetries vanish. Therefore, within the framework of the minimal extension, one has to break S_3 explicitly. To keep the predictivity in the Yukawa sector, we broke it softly. We note that the same soft masses were introduced in [95] to make the heavy Higgses heavy $\gtrsim O(10)$ TeV in order to suppress sufficiently the tree level FCNCs.

(3) Because of $|\sin 2\varphi_3| \lesssim 0.013$, an enhancement is needed. A nice way is the resonant enhancement [17, 18, 22], which requires all the right-handed neutrino masses to be degenerate. [$M_1 = M_2$, up to very small corrections, is ensured by S_3 symmetry even if it is softly broken.]

(4) We also found that the CP asymmetries vanish if the right-handed neutrino masses are much larger than the heavy Higgs masses. Therefore, to obtain a realistic size for baryon asymmetry, we have to assume that the right-handed neutrino masses are bounded from above. The heavy Higgs masses dictate the upper bound.

(5) At last, we investigated the question of how fine the fine tuning needed for the degeneracy of the neutrino masses is. We found that if the criterion (83) on the validity of the approximate formula (61) is satisfied, and the naturalness condition (75) is automatically satisfied. It turned out that the criterion (83) does not depend on the mass of the right-handed neutrinos.

As Fig. 3 presents, it is possible in the present model to explain the observed baryon asymmetry in the universe by leptogenesis. The basic parameters are the right-handed neutrino masses, the heavy Higgs masses and the CP phase. Since the resonant enhancement of CP asymmetries is assumed, and consequently, the degeneracy among the right-handed neutrino masses has to be very precise, it will be very difficult to experimentally determine the right-handed neutrino masses from a precise measurement of baryon asymmetry alone, even if the CP phase is precisely known. Experimentally, this is not a nice feature, but the model predicts (if the observed baryon asymmetry should be explained by leptogenesis) that there will be three extremely degenerate right-handed neutrinos whose masses are comparable with or less than the heavy Higgs masses. On one hand, the smaller the heavy Higgs masses are, the more natural is the fine tuning in the Higgs sector. The masses of the heavy Higgses, on the other hand, are $\gtrsim O(10)$ TeV to sufficiently suppress the tree level FCNCs.

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